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Amount of One or **Future Value of One ($1, £1, ¥1, etc.)**

The future worth of one (unit of money) when invested for a specified period of time with **compound interest**. Given by the formula:

$$A = (1 + i)^n$$

where A is the value of one unit of monetary value invested for n periods of time (or n years) at a compound interest rate of i.

If the interest is compounded more than once during the given periods of time (or more than once per annum), then the amount of one is given by the formula:

$$[1 + (i/m)]mn$$

where m represents the number of times that the interest is credited per period of time (or in a year). Thus, if $1,000 is invested for 8 years at a nominal rate of 9% p.a. and interest is credited to the principal (and compounded) monthly, then it will accumulate to:

$$1{,}000 \times [(1 + (0.09/12)]^{12 \text{ x } 8}$$

= $2,050 at the end of the period.

Present Value (or **Present Worth) of One ($1, £1, ¥1, etc.)**

A sum of money which if invested now at a given rate of **compound interest** will be worth one unit of value at the end of a stipulated period of time (as at the end of a number of months or years).

Given by the formula:

$$V = \frac{1}{(1+i)^n}$$

where: V = the sum invested

I = the interest rate

n = the number of periods of time

This formula may be used to discount a sum of money to take account of its erosion in worth over time. Thus, if $1,000 is receivable in 5 years and is assumed to reduce in 'real value' at the rate of 12 percent per annum, *at present* that sum is worth:

$$1{,}000 \times \{1/[(1 + 0.12)^5]\} = \$567.40$$

Alternatively, if $567.40 is invested today at a compound interest rate of 12%, that sum will be worth $1,000 in 5 years. Called also the payment to amortize 1 unit of value (or $1). In the US, also called a 'reversion factor'.

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Amount of One per Period or **Future Value of One per Period**

The amount to which a *series* of investments, or deposits, of one unit of value will accumulate in a given period (or number of years) at a given rate of compound interest.

Given by the formula:

$$A_n = 1 + (1+i) + (1+i)^2 + (1+i)^3 + \cdots + (1+i)^{n-1}$$

$$= \frac{(1+i)^n - 1}{i}$$

where: A_n = the sum accumulated
i = the interest rate
n = the number of periods of time

Thus, if $10,000 is invested every year for 15 years with interest compounded annually at 6% the amount accumulated will be $10,000 x [(1.0615-1)/0.06] = $232,760.

Called also an 'annuity factor', the 'future annuity of 1 per period' or, sometimes in the US, a 'sinking fund accumulation factor'. Cf. **sinking fund factor**.

Present Value (or **Present Worth**) **of One per Period/Years' Purchase**

The **present value** of a series of future payments, or installments, of one unit of value, that are to be invested at a fixed compound interest rate over a given period of time (or number of years), or the discounted value of a future level income, i.e. of an annuity certain.

Given by the formula:

$$P_n = \frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \frac{1}{(1+i)^{n-1}} \cdots + \frac{1}{(1+i)^n}$$

$$= \frac{1 - \dfrac{1}{(1+i)^n}}{i}$$

where: *Pn* = the value today of a right to receive one unit of value, for *n* periods of time (or years), discounted at an interest rate of *i* per period (or per annum).

Thus, if $1,000 is received every year for 10 years and the resulting sum is assumed to be invested at 8% p.a. for that term, the present value, or single sum, that is equivalent to that income stream is:

$$\left[\frac{1 - \dfrac{1}{(1+0.08)^{10}}}{0.08}\right] = 6.71 \times 1,000 = \$6,710$$

Alternatively, if \$6,710 is invested today at 8%, it would provide an income of \$1,000 per annum for 10 years. Or, if the income from a lease for 10 years is \$1,000 and the appropriate capitalisation rate for that lease is 8%, the lease has a present value of \$6,710.

The present value of one per period is the reciprocal of the amount that will be purchased by an annuity of one unit of value.

In the US, called also a 'present worth factor', an '**Inwood factor**' or 'Inwood coefficient' and in the UK, a **years' purchase**. See also **capitalization factor**, **internal rate of return**, **net present value**.

Annuity One Will Purchase

An annual return, or **annuity**, receivable over a given number of years, from an investment of one unit of value. The sum of money that, if paid in annual installments over the period of a loan, will repay one unit of that loan, together with interest thereon; i.e. an 'annuity factor' that provides a sinking fund to recoup the principal amount of the loan and also repays interest on the outstanding balance.

The factor is given by the formula:

$$a_n = i + S_n = i + \frac{i}{(1+i)^n \quad 1}$$

$$= \frac{i}{1 - \frac{1}{(1+i)^n}}$$

where:

i = interest rate per annum (or per period), expressed as a decimal
S_n = **sinking fund**
n = number of annual (or regular) loan repayments (during the term of the loan)
a_n = the annuity

This annuity figure is most commonly calculated in order to determine the level periodic installments that will amortize a loan, i.e. to calculate a **mortgage constant**. For example, if a loan of \$100,000 is taken out for a period of 25 years, then the constant annual amount required to repay the loan together with interest at 9% per annum is:

$$100{,}000 \text{ x } 0.09/[1-(1.09^{25})^{-1}] = \$10{,}180.63$$

The annuity one will purchase is the reciprocal of the **Inwood factor,** i.e. of the **present value of one per period**.

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Sinking-Fund (s.f.) Factor

A number of equal periodic payments that will accumulate to one unit of value when invested with compound interest; the amount of these payments is calculated by the formula:

$$S_n = \frac{s}{(1+s)^n - 1}$$

where:

S_n = the periodic payment or 'sinking fund factor' or 'sinking fund rate'

s = compound interest rate
n = number of periodic payments.

The sinking fund factor is the reciprocal of the **amount of one per period/ future value of one per period**. Cf. **amortization**. See also **annuity**, **depreciation**, **dual-rate capitalization factor**.

The foregoing formulae may also be expressed as a function of the **Amount of One** or **Future Value** $A^n = (1 + i)^n$ so that:

Present Value of One $= \dfrac{1}{A^n}$

Future Value of One per Period or Amount of One per Period $= \dfrac{A^n - 1}{i}$

Present Value of One per Period Inwood factor or Years' Purchase $= \dfrac{1 - \dfrac{1}{A^n}}{i}$

Annuity One Will Purchase $= i + \dfrac{i}{(A^n - 1)}$

Sinking Fund Factor Sn $= \dfrac{i}{A^n - 1}$

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Capitalisation(BrE) or Capitalization(AmE)

Capitalization for a limited period of time may be expressed by the formula:

$$C_v = \frac{a_1}{(1+r)} + \frac{a_2}{(1+r)^2} + \frac{a_3}{(1+r)^3} + \cdots + \frac{a_n}{(1+r)^n}$$

where:

Cv	=	the capital, or current, value
$a1, a2, \ldots an$	=	the income or cash flow receipts, in each period of time, for n periods.
r	=	capitalization rate, expressed as a decimal.

When the income is constant, 'straight capitalization' produces a capital value = a/r where a = income and r = capitalization rate.

Dual-Rate Capitalization Factor(AmE) or Dual-Rate Years' Purchase(BrE)

A **capitalization factor** or **years' purchase**, used to capitalize income from a depreciating investment (e.g. a leasehold interest or a wasting asset), that incorporates a mathematical adjustment so that the capital value obtained is comparable to a similar, but non-depreciating, investment. In the case of a leasehold investment, by providing a sinking fund the investor is able to regard the investment as perpetual. The capital value obtained by applying this factor to a projected income from an investment is such that the investor receives both (i) a **remunerative rate** of return which is comparable to a permanent investment; and (ii) a notional **sinking fund**, which is set aside at a 'safe' or **accumulative rate** of return, to replace the original cost of the investment at the end of its useful or anticipated life. A dual-rate factor is 'adjusted' to notionally set aside income from the investment to replace the original capital cost, with the result that the capital value of the declining income is reduced so that the net return to the investor is comparable with the level achieved by an investor who acquires a permanent, or non-depreciating, asset. The factor may be calculated by the formula:

$$r = \frac{1}{\text{interest rate} + \text{sinking fund factor}}$$

$$\text{or} \quad \frac{1}{(i + S_n)}$$

$$= \frac{1}{i + \frac{s}{(1+s)^n - 1}}$$

where:

r	=	capitalization factor
i	=	remunerative rate
S	=	sinking fund factor
s	=	accumulative or sinking fund rate
n	=	number of income receipts during the term of the investment (assumed in arrears).

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Thus, if a wasting asset is acquired for P and produces an income of v per annum, such that $P = rv$, then the investor will receive a return on his investment of i per annum, which is less than v throughout the term of n years, with the difference between v and i notionally invested at a rate of interest s in order to accumulate to capital sum P to replace the wasted asset; in other words, hypothetically, to perpetuate the income. (With a single-rate **capitalization factor** there is a notional sinking fund, but it is deemed to accumulate at the same rate as the remunerative rate, i.e. $i = s$). The dual-rate capitalization formula only provides for the replacement of the original capital cost and makes no allowance for inflation or tax on the sinking fund element. In the US, this factor is called also the '**Hoskold factor**', or this method of capitalizing income the 'Hoskold approach' or the 'Hoskold premise'. See also **internal rate of return**.

W. Britton, K. Davies and T. Johnson. *Modern Methods of Valuation* (8th ed. London: 1989), pp. 110-117.

Net Present Value (NPV)

The difference, at a given discount rate, between the **present value** of (i) the total net income to be derived from an investment, and (ii) the total expenditure and outgoings incurred in making and maintaining that investment, taken over the projected life of the investment. Net present value is used especially to compare or to assess the viability of alternative investments. The income and expenditure from each alternative is reduced or discounted to a common base, i.e. a single value is determined by discounting to the present day (or the date of the initial investment) the amounts receivable over and above the amounts payable for each investment. A positive NPV indicates that an investment is profitable, and the higher the NPV, the better the profitability of that investment. The net present value of the **net income** receivable from an investment is equivalent to the **capital value** of that investment and may be obtained from the formula:

$$NPV = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t} = CF_0 + \frac{CF_1}{(1+i)} + \frac{CF_2}{(1+i)^2} + \frac{CF_3}{(1+i)^3} \cdots + \frac{CF_n}{(1+i)^n}$$

where: CF_0 CF_1 CF_2 CF_3 CF_n = net cash flows from the investment at each period of time

i = discount rate

n = number of periods.

If $NPV = 0$, then i = the **internal rate of return**.

In other words, NPV represents an 'absolute' measure of value, whereas IRR is a 'relative' rate of return.

Also called the 'net discounted value' or the 'net discounted revenue'. See also **capitalization**, **discounted cash flow**.

G. Brown & G.A. Matysksak. *Real Estate Investments: A Capital Market Approach* (Harlow, Essex: 2000), pp. 154-157.

W.D. Fraser. *Cash-Flow Appraisal for Property Investment* (Basingstoke, Hants: 2004), Ch. 4 'Understanding IRR and NPV'.

S. Lumby & C. Jones. *Investment Appraisal and Financial Decisions* (6th ed. 1999), Ch. 6 'Net present value and internal rate of return'.

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Internal Rate of Return (IRR)

The rate of interest that discounts a series of future cash flows or income returns to make them equal to the total cost or outlay on the investment that generates those cash flows or income returns; the one rate of interest at which the **present value** of all expenditure on an investment equals the present value of all receipts from that investment (i.e. the discount rate when the **net present value** is zero). An internal rate of return considers the time at which the cash flow is received as well as the total sum of the cash flows. It was called the 'rate of return over cost' by Irving Fisher, and 'the marginal efficiency of capital' by J.M. Keynes; "that rate of discount which should discount the value of a series of annuities, given by the return from the capital asset during its life, just equal to its supply price", *The General Theory of Employment, Interest and Money* (1936), p. 135.

The internal rate of return may be calculated by solving for r in the formula:

$$P_0 = \sum_{1=n}^{i=n} \frac{R_i}{(1+r)} + \frac{P_n}{(1+r)^n}$$

where:

P_0 = initial cost or 'supply price'

R_i = income during period i (or per annum), in arrears

P_n = value of reversion or **redemption value** in period n (or the scrap value after n years)

n = number of periods (or years)

r = internal rate of return

Thus, if P_0 is the price paid for an investment, which produces a periodic (or annual) income in arrears of R_i for n periods (or years), and the investment is sold at the end of that period for P_n then the equivalent annual return over the life of the investment is r. Under this formula, it is assumed that all income received from the investment is reinvested during the term of the investment at the same rate. The rate calculation may be 'adjusted' by applying a lower or safer rate to the income received throughout the term of the investment.

The internal rate of return is used to make comparisons between alternative investments; to decide if the return is sufficient to warrant parting with money; or to decide if it is worth borrowing money at a given rate of interest in order to make an investment. It is a measure of the inducement to invest because it measures the return of capital, i.e. it is a 'derived return'. It may be contrasted with the rate of interest incorporated in a **capitalisation factor** (years' purchase or Inwood factor) which is an 'applied return', i.e. one used to ascertain the capital value of an investment, given the interest rate or the cost of parting with or borrowing money.

The IRR may be 'modified' by discounting any negative cash flows (expenses) at a specified safe rate (a lower rate) and the positive income is discounted at the normal investment rate. This 'modified internal rate of return' is also called an 'adjusted rate of return'. If the calculation uses different rates of return for reinvestment of income at different stages of the holding period (as when part of the income is held in a deposit account and not assumed to be reinvested in the primary investment) the result is called the 'financial management rate of return'. The internal rate of return may be calculated at a given point in time, but assuming the investment is sold at a series of future dates, such as at the end of year 1, year

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2, year 3, etc;called a 'rolling IRR'.
Also called the 'discounted rate of return' or 'discounted cash flow rate of return'; the 'actuarial investment return', the 'overall rate of return'; the 'investment rate of return' or sometimes the 'investor's rate of return' or 'true rate of return'. See also **discounted cash flow**, **redemption yield**.

C.B. Akerson. The Internal Rate of Return in Real Estate Investment (rev. ed. 1976).
W.D. Fraser. *Cash-Flow Appraisal for Property Investment* (Basingstoke, Hants: 2004), Ch. 4 'Understanding IRR and NPV'.
A.J. Jaffe & C.F. Sirmans. Fundamentals of Real Estate Investment (3rd ed. 1995), pp. 334-335, 349-350.

equivalent yield (BrE)

The single yield from an investment property taking into account the total income receivable from the property, including any increases in rent due to rent revisions. The equivalent yield is based on current values; thus, if the property is rented at the current rack rent, the nominal yield is the same as the equivalent yield. When there is a single rent review, the equivalent yield on an investment may be obtained from the formula:

$$P_0 = \frac{r}{e} + \frac{(R - r)}{e(1+e)^n}$$

where:
- P_0 = initial investment or cost
- r = initial rental income
- R = estimated market rental value
- n = number of years to the next market rent review
- e = equivalent yield.

If the rent is receivable at regular, but not annual, intervals in advance (such as quarterly or monthly in advance) the yield taking into account this timing is referred to as the 'true equivalent yield'. The 'equivalent yield' may be contrasted with the 'nominal yield' which is calculated based on the direct ratio of the total annual rent to the capital value of the investment. See also **equated yield**.

N. Enever & D. Isaac. *The Valuation of Property Investments* (6th ed. London: 2002), pp. 136-139.

effective annual interest rate

The true interest rate payable over one year when interest is compounded at intervals of less than a year. The formula for converting a **nominal interest rate** to an effective annual rate is:

$$I = [1 + \frac{i}{m}]^m - 1$$

where:
- i = nominal interest rate (expressed as a decimal)
- m = number of compounding periods per annum
- I = effective annual interest rate.

Thus, a nominal interest rate of 8% per annum would be an effective rate of 8.3% when interest is compounded monthly. Also called an 'effective rate' or, when the same conversion is made to arrive at the rate of return on an investment that produces an income at intervals of less than once a year, the 'effective rate of return'. See also **annual percentage rate**, **internal rate of return**.

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constant-rent factor(BrE)

A factor that may be applied to the rent payable in the open market to take account of the benefit of a longer than normal interval between one rent review and the next. Thus, if a tenant has a lease with 14-year intervals between reviews of the market rent and the accepted pattern of rent reviews is 5 years, then a tenant is likely to be prepared to pay a different rent at the outset because his initial rent is fixed for a longer period of time: a higher rent if he anticipates rising rental values and a lower rent if he anticipates falling rental values. This differential is represented by the 'constant-rent factor', which can be obtained using the following formula:

$$\frac{(1+r)^n - (1+g)^n}{(1+r)^n - 1} \times \frac{(1+r)^t - 1}{(1+r)^t - (1+g)^t}$$

where: r = lessor's required return on capital
g = annual rate of rental growth (or decline)
n = number of years between rent reviews in the actual lease
t = number of years between normal rent reviews.

This formula was devised by Jack Rose in 1979 and may also be called an 'uplift factor' (assuming that rents are expected to rise). If this formula is applied to the market rent payable under a lease with conventional rent review periods, it produces the 'equated rent' that the tenant may be prepared to pay at the start of the lease. See also **interim rent**.

J.J. Rose. *Tables of the Constant Rent* (Oxford: 1979).

Bibliography

Australia

Australian Property Institute. *Valuation Principals and Practice* (2nd ed. Deakin, ACT: 2007), Ch. 14 'Financial Mathematics'.

Whipple, R.M.T. *Property Valuation and Analysis*, 2nd ed. Pyrmont, NSW: 2006, Chs. 7 & 8 'Financial Analysis'.

United Kingdom

J. Armatys et al. *Principles of Valuation* (London: 2009), Ch. 13 'The Time Value of Money and Valuation Tables'.

Bowcock, Philip and Natalie Bayfield. *Advanced Excel for Surveyors*. London: Estates Gazette, 2000.

Bowcock, Philip. *Excel for Surveyors*. London: Estates Gazette, 2000.

Bowcock, Philip. *Property Valuation Tables*. London: Macmillan, 1978.

Brown, Gerald R. and George A. Matysksak. *Real Estate Investments: A Capital Market Approach*. Harlow, Essex: Pearson Education, 2000.

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Johnson, Tony, Keith Davies and Eric Shapiro. *Modern Methods of Valuation*, 9th ed. Ch. 8 'The Mathematics of Valuation Tables'. London: Estates Gazette, 2000.

Davidson, A.W. *Parry's Valuation and Investment Tables*, 12th ed. London: Estates Gazette, 2002.

Enever, Nigel and David Issac. *The Valuation of Property Investments*, 6th ed. London: Estates Gazette, 2002, Ch. 6. 'The Mathematics of Valuation and Finance'.

Fawcett, Scott. *Designing Flexible Cashflows*. London: Estates Gazette, 2003.

P. Marshall. *Donaldsons Investment Tables*. London: Donaldsons, 1979.

Rose's Property Valuation Tables, constituting the 34th edition of INWOOD'S TABLES, reconstructed and explained by J.J. Rose. Oxford: The Freeland Press Ltd., 1975.

Royal Institution of Chartered Surveyors. *Commercial Investment Property: Valuation Methods, An Information Paper*, London: RICS Books, 1997, Ch. 2 'Appraisal Mathematics'.

United States

Akerson, Charles B. *Capitalization Theory and Techniques: Study Guide (with financial tables, including J and K factor tables)*, 2nd ed. Chicago: Appraisal Institute, 2000.

AIREA. *Financial Tables, edited and compiled by James J. Mason.* Chicago: Appraisal Institute, 1981.

Appraisal Institute. *The Appraisal of Real Estate*, 13th ed. Chicago: Appraisal Institute, 2008, Ch. 13 'Direct Capitalization', Ch. 14 'Yield Capitalization—Theory and Basic Applications'.

Appraisal Institute. *The Appraisal of Real Estate*, 13th ed. Chicago: Appraisal Institute, 2008, Appendix B 'Financial Formulas'.

Ellwood, L.W. *Ellwood Tables for Real Estate Appraising and Financing,* 4th ed. Cambridge, MA; Ballinger Publishing Co., 1977.

Fisher, Charles E. *Mathematics of Real Estate Appraisers: an Appraisal Institute Handbook.* Chicago: Appraisal Institute, 1996.

Geltner, D. et al. *Commercial Real Estate Analysis & Investment,* 2nd ed. Florence, KY: South-Western Cengage Learning, 2007, with CD-ROM. Ch. 9 'Measuring Investment Performance: The Concept of Return', Pt. IV 'Real Estate Valuation and Investment Analysis', Pt V 'Completing the Basic Investment Analysis Picture'.

Sindt, Roger P. *Real Estate Investment: Analysis and Applications*, Upper Saddle River, NJ: Prentice-Hall, 1997, Ch. 3 'Real Estate Investment Mathematics'.

Wiley, Robert J. *Real Estate Accounting and Mathematics Handbook*, 3rd ed. New York: Wiley, 1993.

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